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AVERAGE AND PROBABILITY.

154. Proposed by L. C. WALKER, A. M., Santa Barbara, Cal.

Five points are taken at random in space. The chance that the fifth point may be included within the tetrahedron of which the other four points are the vertices is $\frac{1}{27}$.

Solution by the PROPOSER.

With the five points taken at random as vertices, five tetrahedrons may be formed.

Let A, B, C, D be the four points which form the greatest tetrahedron, and let E be the fifth point. Through the vertices A, B, C, D, pass the planes $B'C'D'$, $A'C'D'$, $A'B'D'$, $A'B'C'$, parallel, respectively, to the planes BCD, ACD, ABD, ABC. Now E must fall within the tetrahedron $D-ABC$; for if E fell without it, the tetrahedron $D-ABC$ would not be the greatest tetrahedron which could be formed with the five random points as vertices.

Consequently the volume of the tetrahedron $D'-A'B'C'$ represents the extent of all possible cases; that is, of all cases consistent with the assumption that $D-ABC$ is the greatest tetrahedron that could be formed with the five random points as vertices. The volume of the tetrahedron $D-ABC$ represents the total number of favorable cases. Hence the required chance is

$$\Delta=\frac{\text{the volume of tetrahedron } D-ABC}{\text{the volume of tetrahedron } D'-A'B'C'}=\tfrac{1}{27}.$$

156. Proposed by J. E. SANDERS, Hackney, Ohio.

Find the average area of a triangle, the sum of whose sides is constant and equal to $2a$.

Solution by the PROPOSER.

Let x, y, $2a-x-y$ be the sides.

Then the area is $A=\surd[a(a-x)(a-y)(x+y-a)]$.

The limits of x are 0 and a; of y, $a-x$ and a. Putting Δ for the average area required, we have

$$\Delta=\int_0^a\int_{a-x}^a Adxdy \Big/ \int_0^a\int_{a-x}^a dxdy=\frac{2}{a^2}\int_0^a\int_{a-x}^a Adxdy.$$

Let $a-y=u$, $du=-dy$; the limits of u are x and 0.

$$\therefore\ \Delta=\frac{2}{a^2}\int_0^a\int_0^x \surd[a(a-x)(xu-u^2)]dxdu=\frac{\pi}{4a^2}\int_0^a x^2\surd[a(a-x)]dx=\tfrac{4}{105}\pi a^2.$$

Solved with the same result by S. A. Corey, and G. B. M. Zerr.